IUHET 280
YCTP-P5-94
hep-ph/9409404
June 1994

NUCLEAR NULL TESTS FOR SPACELIKE NEUTRINOS

Alan Chodos[a] and V. Alan Kostelecký[b]

[a]*Physics Department*
Yale University
New Haven, CT 06511, U.S.A.

[b]*Physics Department*
Indiana University
Bloomington, IN 47405, U.S.A.

Recently, a type of null experiment for spacelike neutrinos has been proposed. We examine in detail a class of null tests involving nuclear beta decay or capture in atoms and ions. The most promising candidate systems are identified.

Accepted for publication in Physics Letters B

1. Introduction

Recently, we proposed a class of null experiments designed to test whether the four-momentum of either the electron neutrino or the muon neutrino might in fact be spacelike [1, 2, 3]. Originally stimulated by theoretical considerations, this unconventional idea has received further examination in light of the ongoing trend towards negative values in the measurement of squared neutrino masses. Five recent experiments [4, 5, 6, 7, 8] have measured the squared mass of the electron neutrino to be negative, with the mean more than two standard deviations from zero. Measurements of the squared mass of the muon neutrino over the past decade [9, 10, 11, 12, 13] have similarly produced negative values, with the latest being five standard deviations from zero. The situation in the muonic case has been altered very recently by a remeasurement of a crucial pionic x-ray intensity, which allows a new solution for the pion mass that yields a muon-neutrino squared mass compatible with zero [14]. An independent confirmation of this possibility would be very interesting. In any event, the issue is ultimately experimental. With this in mind, we present in this letter an analysis of a class of possible experiments that can provide a *lower* (negative) bound on the squared mass of the electron neutrino.

Although the focus in this paper is on the electron neutrino, the basic idea is most easily illustrated for the muon neutrino, which we summarize first. More details about the theoretical motivation and a variety of null tests are presented in [1, 2, 3].

Consider the process

$$\mu \to \pi + \nu_\mu \quad . \tag{1}$$

When the muon is at rest, this reaction is forbidden by energy conservation.[1] If the neutrino has spacelike momentum, however, it is possible to boost the muon to a frame in which the process is kinematically allowed. The threshold for the reaction to occur is given by the muon energy

$$E_{\rm th} \;=\; \frac{1}{2|m_\nu|}\left[(m_\pi^2 - m_\mu^2)^2 + 2(m_\pi^2 + m_\mu^2)m_\nu^2 + m_\nu^4\right]^{1/2}$$

[1]We disregard here the possibility of negative-energy neutrinos, which leads to a variety of disagreements with observation.

$$\simeq \frac{4.2 \times 10^3}{|m_\nu/\mathrm{MeV}|} \quad \mathrm{MeV} \quad , \tag{2}$$

where m_ν is defined to satisfy $m_\nu^2 = p_\nu^2 - E_\nu^2$. A crude estimate of the rate ω of the reaction (1) can be obtained under a number of theoretical assumptions [2]. It exhibits a peak at $E_{\max} = \sqrt{3}E_{\mathrm{th}}$, varying with m_ν and $\bar{Q} = m_\pi - m_\mu$ as

$$\omega(E_{\max}) \propto \frac{m_\nu^3}{\bar{Q}} \quad , \tag{3}$$

which makes the rate small. Nevertheless, it can be combined with experimental data on muon beams to place a *lower* (negative) bound on the squared mass of the muon neutrino [3].

Similar considerations also apply to processes involving the electron neutrino. For example, certain β decays forbidden kinematically when the decaying nucleus is at rest could be allowed if the nucleus were boosted. More stringent limits have been placed on the mass of the electron neutrino, so the corresponding threshold energy E_{th} might be expected to be higher, and the corresponding rate $\omega(E_{\max})$ smaller. However, as $m_\mu \to m_\pi$ for fixed m_ν, Eqs. (2) and (3) reveal that the threshold energy becomes independent of m_ν while the small factor m_ν^3 in the numerator of the rate can be overcome by the shrinking value of $\bar{Q}$. In other words, if the barrier to the decay at rest is decreased, then the threshold energy is reduced while the decay rate is enhanced.

These considerations suggest it is worthwhile to pursue a search for nuclear β processes forbidden kinematically but for which the magnitude $\bar{Q}$ of the associated negative Q value is very small. This paper presents the results of such an investigation and thereby identifies the most promising candidates for future experiments.

To ensure a comprehensive search, it is necessary to consider not only the usual beta decay involving electrons and antineutrinos, but also processes like positron emission and electron capture. Moreover, since a nuclear null experiment is likely to involve the acceleration of the nucleus in question, some or all of the electrons may need to be stripped from the neutral atom. This procedure can have a profound effect on the Q value for the reaction. For example, consider the beta decay of the neutral atom ${}^{163}_{66}\mathrm{Dy}$, via the process ${}^{163}_{66}\mathrm{Dy} \to {}^{163}_{67}\mathrm{Ho} + \bar{\nu}_e$. This reaction, which converts one

neutral atom to another, has a small negative Q value, $Q \simeq -2.6$ keV. However, the Q value for beta decay of the fully stripped dysprosium nucleus into a holmium ion with one bound electron is *positive,* $Q \simeq +50$ keV. Indeed, this reaction has recently been experimentally observed [15]. The difference in behavior between the neutral atom and the fully stripped nucleus suggests an examination of intermediate, partially stripped ions is needed to identify the most promising candidate reactions for nuclear null tests.

2. Analysis and Results

To investigate these questions systematically, we begin by introducing a more complete notation. We use the symbol ${}^{A}_{Z}\mathrm{N}(k)$ to denote a nucleus of element N with baryon number A, charge Z, and with k electrons bound to it. The lifetime of the corresponding neutral atom is denoted by τ_{N}. The ionicity $n = Z - k$ is defined as the net positive charge on the ion. In what follows, we restrict our attention to the case $0 \le k \le Z$, i.e., we disregard the possibility of negative ions.

Our purpose requires a study of the energetics of any given process. For simplicity, the symbol ${}^{A}_{Z}\mathrm{N}(k)$ is also used to denote the rest energy of a given ion. The absolute value of the binding energy of k electrons to a nucleus of charge Z is denoted by $B(k, Z)$. We then immediately obtain

$$\begin{aligned} {}^{A}_{Z}\mathrm{N}(k) &= {}^{A}_{Z}\mathrm{N}(0) + k m_e - B(k, Z) \\ &= {}^{A}_{Z}\mathrm{N}(Z) - n m_e + B(Z, Z) - B(k, Z) \quad . \end{aligned} \tag{4}$$

These two formulae relate ${}^{A}_{Z}\mathrm{N}(k)$ either to the mass ${}^{A}_{Z}\mathrm{N}(0)$ of the bare nucleus or to the rest energy ${}^{A}_{Z}\mathrm{N}(Z)$ of the neutral atom.

We begin the analysis with a consideration of processes involving two-body final states. There are two types of such processes to discuss. First, we examine reactions of the type

$${}^{A}_{Z}\mathrm{N}_1(k) \to {}_{Z+1}^{\;\;\,A}\mathrm{N}_2(k+1) + \bar{\nu}_e \quad \text{(type 2a)} \quad , \tag{5}$$

in which the nucleus N_1 undergoes beta decay and the emitted electron is captured by the ensuing ion N_2. Using Eq. (4), the Q values $Q_{2a}(k)$ for this class of processes

${}^{A}_{Z}\mathrm{N}_1$	τ_{N_1}	${}^{A}_{Z+1}\mathrm{N}_2$	$Q_{2a}(Z)$ (keV)
${}^{163}_{66}$ Dy	—	${}^{163}_{67}$ Ho	-2.6
${}^{243}_{95}$ Am	7.4×10^3 y	${}^{243}_{96}$ Cm	-7.3
${}^{202}_{81}$ Tl	12.2 d	${}^{202}_{82}$ Pb	-46.0
${}^{194}_{79}$ Au	38.0 h	${}^{194}_{80}$ Hg	-50.0
${}^{123}_{51}$ Sb	—	${}^{123}_{52}$ Te	-52.0
${}^{157}_{64}$ Gd	—	${}^{157}_{65}$ Tb	-57.6
${}^{205}_{81}$ Tl	—	${}^{205}_{82}$ Pb	-60.0
${}^{136}_{54}$ Xe	—	${}^{136}_{55}$ Cs	-67.0
${}^{213}_{84}$ Po	4 μs	${}^{213}_{85}$ At	-74.0
${}^{244}_{94}$ Pu	8.1×10^7 y	${}^{244}_{95}$ Am	-76.0

Table 1: Smallest negative $Q_{2a}(Z)$ values for two-body reactions of type 2a.

can be expressed as

$$\begin{aligned} Q_{2a}(k) &= {}^{A}_{Z}\mathrm{N}_1(Z) - {}^{A}_{Z+1}\mathrm{N}_2(Z) + B(Z,Z) - B(k,Z) \\ &\quad -B(Z+1,Z+1) + B(k+1,Z+1) \\ &= {}^{A}_{Z}\Delta - \Delta B_Z + \Delta B_k \quad . \end{aligned} \tag{6}$$

In the second form of this equation, the so-called beta-decay energy ${}^{A}_{Z}\Delta$ is given by

$${}^{A}_{Z}\Delta = {}^{A}_{Z}\mathrm{N}_1(Z) - {}^{A}_{Z+1}\mathrm{N}_2(Z+1) \quad . \tag{7}$$

The remaining factors are defined by

$$\Delta B_k = B(k+1, Z+1) - B(k,Z) \quad , \tag{8}$$

which is positive for all k. The quantity $\Delta B_Z \equiv \Delta B_{k=Z}$ is the difference in total electronic binding energy between the two neutral atoms.

Let us first consider type-2a reactions with $k = Z$, i.e., those involving neutral atoms. For this case, $\Delta B_Z = \Delta B_k$ so we find $Q_{2a}(Z) = {}^{A}_{Z}\Delta$. Values of ${}^{A}_{Z}\Delta$ can be obtained from a table of atomic masses. Table 1 shows the most favorable cases extracted from the compilation in ref. [16]. Since some of the candidate atoms are unstable, we also list their lifetimes τ_{N_1} taken from ref. [17].

The values of $Q_{2a}(k)$ for other nonzero k, i.e., the partially stripped cases, are not readily found. The problem is that, to our knowledge, there is no compendium of

${}^A_Z$N$_1$	τ_{N_1}	${}^{\ A}_{Z+1}$N$_2$	${}^A_Z\Delta$	ΔB_Z	$B(1, Z+1)$	$Q_{2a}(0)$
${}^{157}_{64}$Gd	—	${}^{157}_{65}$Tb	-57.6	11.9	61.0	- 8.5
${}^{235}_{92}$U	7.0×10^8 y	${}^{235}_{93}$Np	-123.0	21.6	135.2	- 9.4
${}^{123}_{51}$Sb	—	${}^{123}_{52}$Te	-52.0	8.5	38.2	-22.3
${}^{178}_{73}$Ta	2.4 h	${}^{178}_{74}$W	-89.0	14.7	80.8	-22.9
${}^{238}_{92}$U	4.5×10^9 y	${}^{238}_{93}$Np	-145.6	21.6	135.2	-32.0
${}^{136}_{54}$Xe	—	${}^{136}_{55}$Cs	-67.0	9.2	42.9	-33.3
${}^{176}_{70}$Yb	—	${}^{176}_{71}$Lu	-109.6	13.7	73.8	-49.6
${}^{179}_{72}$Hf	—	${}^{179}_{73}$Ta	-115.0	14.3	78.3	-51.0
${}^{160}_{64}$Gd	—	${}^{160}_{65}$Tb	-102.3	11.9	61.1	-53.2
${}^{82}_{34}$Se	—	${}^{82}_{35}$Br	-88.0	4.8	16.9	-75.9

Table 2: Smallest negative $Q_{2a}(0)$ values in keV for two-body reactions of type 2a.

data permitting a determination of values of ΔB_k for nonzero k. Resolving this issue requires an alternative approach, to which we return below.

Instead, consider the case $k = 0$. This corresponds to the beta decay of a fully stripped nucleus into an ion with one bound electron. The Q value for a process of this type is given by $Q_{2a}(0) = {}^A_Z\Delta - \Delta B_Z + B(1, Z+1)$, since by definition $B(0, Z) \equiv 0$. The quantity ΔB_Z can be calculated from tables of total atomic energies [18], while the quantity $B(1, Z+1)$ is provided in ref. [19]. We have determined the candidate processes with the most favorable $Q_{2a}(0)$ values from among the approximately 1500 nuclides for which data are available. These processes are listed in Table 2.

Next, we consider the second type of process with a two-body final state, namely, electron capture ('inverse beta decay'). Typically, this proceeds from the K shell, but L- and higher-shell captures can also occur. The latter are suppressed by wavefunction overlap but are energetically favored. Unless an experiment clearly determines the capture type, the energetics of a null test require the assumption that the outermost electron is captured. We therefore consider reactions of the form

$$ {}^{\ A}_{Z+1}\mathrm{N}_2(k+1) \to {}^A_Z\mathrm{N}_1(k) + \nu_e \quad \text{(type 2b)} \quad . \tag{9}$$

We denote the corresponding Q values by $Q_{2b}(k+1)$, and using Eqs. (4) and (6) we find

$$ Q_{2b}(k+1) = -Q_{2a}(k) \quad . \tag{10}$$

${}^{\ A}_{Z+1}\mathrm{N}_2$	τ_{N_2}	${}^{A}_{Z}\mathrm{N}_1$	$Q_{2b}(Z+1)$ (keV)
${}^{187}_{76}$ Os	—	${}^{187}_{75}$ Re	-2.6
${}^{2}_{3}$ He	—	${}^{1}_{3}$ H	-18.6
${}^{241}_{95}$ Am	433 y	${}^{241}_{94}$ Pu	-20.8
${}^{222}_{87}$ Fr	14.2 m	${}^{222}_{86}$ Rn	-32.0
${}^{148}_{64}$ Gd	75 y	${}^{148}_{63}$ Eu	-33.0
${}^{107}_{47}$ Ag	—	${}^{107}_{46}$ Pd	-33.1
${}^{250}_{97}$ Bk	3.2 h	${}^{250}_{96}$ Cm	-37.0
${}^{106}_{45}$ Rh	29.8 s	${}^{106}_{44}$ Ru	-39.4

Table 3: Smallest negative $Q_{2b}(Z+1)$ values for two-body reactions of type 2b.

${}^{\ A}_{Z+1}\mathrm{N}_2$	τ_{N_2}	${}^{A}_{Z}\mathrm{N}_1$	${}^{A}_{Z}\Delta$	ΔB_Z	$B(1,Z+1)$	$Q_{2b}(1)$
${}^{215}_{86}$Rn	2.3 μs	${}^{215}_{85}$At	-82.0	18.8	112.8	-12.1
${}^{213}_{85}$At	0.11 μs	${}^{213}_{84}$Po	-74.0	18.4	109.9	-17.5
${}^{3}_{2}$He	—	${}^{3}_{1}$H	18.6	0.0	0.0	-18.6
${}^{205}_{82}$Pb	1.5×10^{7} y	${}^{205}_{81}$Tl	-60.0	17.4	101.3	-24.0
${}^{194}_{80}$Hg	520 y	${}^{194}_{79}$Au	-50.0	16.7	95.9	-29.2
${}^{202}_{82}$Pb	5.3×10^{4} y	${}^{202}_{81}$Tl	-46.0	17.4	101.3	-38.0
${}^{246}_{98}$Cf	35.7 h	${}^{246}_{97}$Bk	-80.0	23.8	153.1	-49.4
${}^{163}_{67}$Ho	4.6×10^{3} y	${}^{163}_{66}$Dy	-2.6	12.5	65.1	-50.1

Table 4: Smallest negative $Q_{2b}(1)$ values in keV for two-body reactions of type 2b.

Just as for the case of type-2a processes, available data permit a numerical study of reactions with neutral atoms, $k=Z$, and with fully stripped nuclei, $k=0$. Since the right-hand side of Eq. (10) has a negative sign, it suffices to repeat the previous procedures but keeping now processes with small *positive* ${}^{A}_{Z}\Delta$ listed in ref. [16]. The most favorable cases for $Q_{2b}(Z+1)$ and $Q_{2b}(1)$ are displayed in Tables 3 and 4, respectively.

We next turn to a consideration of processes involving three-body final states. There are again two types of such processes to examine. We begin with reactions of the standard beta-decay type

$$ {}^{A}_{Z}\mathrm{N}_1(k) \to {}^{\ A}_{Z+1}\mathrm{N}_2(k) + e^- + \bar{\nu}_e \quad \text{(type 3a)} \quad . \tag{11}$$

The Q values $Q_{3a}(k)$ for processes of this type can be expressed using Eq. (4) as

${}^{A}_{Z}\mathrm{N}_1$	τ_{N_1}	${}_{Z+1}^{\;\;A}\mathrm{N}_2$	${}^{A}_{Z}\Delta$	ΔB_Z	$\Delta\widehat{B}_1$	$Q_{3a}(1)$	$Q_{3a}(0)$
${}^{241}_{94}\mathrm{Pu}$	14.4 y	${}^{241}_{95}\mathrm{Am}$	20.8	22.4	3.5	> 0	-1.6
${}^{187}_{75}\mathrm{Re}$	4.4×10^{10} y	${}^{187}_{76}\mathrm{Os}$	2.6	15.3	2.5	-10.2	-12.7
${}^{163}_{66}\mathrm{Dy}$	—	${}^{163}_{67}\mathrm{Ho}$	-2.6	12.5	2.0	-13.1	-15.1
${}^{243}_{95}\mathrm{Am}$	7.4×10^{3} y	${}^{243}_{96}\mathrm{Cm}$	-7.3	22.9	3.6	-26.5	-30.1
${}^{202}_{81}\mathrm{Tl}$	12.2 d	${}^{202}_{82}\mathrm{Pb}$	-46.0	17.4	2.7	-60.7	-63.4

Table 5: Binding energies and smallest $Q_{3a}(1)$ and $Q_{3a}(0)$ values in keV for three-body reactions of type 3a.

$$\begin{aligned} Q_{3a}(k) &= {}^{A}_{Z}\mathrm{N}_1(Z) - {}_{Z+1}^{\;\;A}\mathrm{N}_2(Z+1) + B(Z,Z) - B(k,Z) \\ &\qquad -B(Z+1,Z+1) + B(k,Z+1) \\ &= {}^{A}_{Z}\Delta - \Delta B_Z + \Delta\widehat{B}_k \quad , \end{aligned} \tag{12}$$

where $\Delta\widehat{B}_k = B(k, Z+1) - B(k, Z)$ is a non-negative quantity.

There are several cases of type-3a processes accessible to analysis. For reactions involving an initially neutral atom, $k = Z$, the quantity $-\Delta B_Z + \Delta\widehat{B}_k$ reduces to

$$-\Delta B_Z + \Delta\widehat{B}_Z \equiv -I(Z) = -B(Z+1, Z+1) + B(Z, Z+1) \quad , \tag{13}$$

where $I(Z)$ is just the (positive) first-ionization energy of the nuclide N_2. The order of magnitude of $I(Z)$ is one to ten eV, so to an excellent approximation we can write $Q_{3a}(Z) \approx {}^{A}_{Z}\Delta = Q_{2a}(Z)$. A listing of the most favorable such processes has already been given in Table 1.

As mentioned above for the two-body processes of type 2a, we know of no tabulation of data that permit a determination of $Q_{3a}(k)$ for arbitrary nonzero k. However, the situation is more favorable than for type-2a processes because Eq. (12) involves $\Delta\widehat{B}_k$ rather than ΔB_k. The data compiled in ref. [19] therefore permit an analysis of the case where $k = 1$. The situation for the fully stripped nucleus with $k = 0$, which has $\Delta\widehat{B}_k = 0$, can also be analyzed. Among the nuclides for which information is available, we have found only a few candidates of interest for either $k = 0$ or $k = 1$. They are listed in Table 5. Whenever Q is negative, the candidate with $k = 1$ is more favorable than the corresponding one for $k = 0$ because $\Delta\widehat{B}_1$ is positive.

The second class of reaction involving a three-body final state is positron emission,

${}^{A}_{Z+1}\mathrm{N}_2$	τ_{N_2}	${}^{A}_{Z}\mathrm{N}_1$	${}^{A}_{Z}\Delta$ (keV)	$Q_{3b}(Z)$ (keV)
${}^{185}_{76}\mathrm{Os}$	93.6 d	${}^{185}_{75}\mathrm{Re}$	-1015.0	-7.0
${}^{67}_{31}\mathrm{Ga}$	3.3 d	${}^{67}_{30}\mathrm{Zn}$	-1001.1	-20.9
${}^{122}_{54}\mathrm{Xe}$	20.1 h	${}^{122}_{53}\mathrm{I}$	-1000.0	-22.0
${}^{191}_{78}\mathrm{Pt}$	2.9 d	${}^{191}_{77}\mathrm{Ir}$	-1000.0	-22.0
${}^{83}_{37}\mathrm{Rb}$	86.2 d	${}^{83}_{36}\mathrm{Kr}$	-998.0	-24.0
${}^{236}_{93}\mathrm{Np}$	1.2×10^5 y	${}^{236}_{92}\mathrm{U}$	-984.0	-38.0
${}^{238}_{96}\mathrm{Cm}$	2.4 h	${}^{238}_{95}\mathrm{Am}$	-980.0	-42.0
${}^{203}_{82}\mathrm{Pb}$	52.0 h	${}^{203}_{81}\mathrm{Tl}$	-974.0	-48.0

Table 6: Smallest negative $Q_{3b}(Z)$ values for three-body reactions of type 3b.

with the generic form

$$ {}^{A}_{Z+1}\mathrm{N}_2(k) \rightarrow {}^{A}_{Z}\mathrm{N}_1(k) + e^+ + \nu_e \quad \text{(type 3b)} \quad . \tag{14} $$

We find the associated Q value $Q_{3b}(k)$ is related to $Q_{3a}(k)$ as given in Eq. (12) by

$$ Q_{3b}(k) = -Q_{3a}(k) - 2m_e \quad . \tag{15} $$

As for type-3a processes, the data available make possible an investigation of positron emission for the cases with $k = Z$, $k = 1$, and $k = 0$. The presence of the factor of $-2m_e$ on the right-hand side of Eq. (15) means that reactions of most interest have ${}^{A}_{Z}\Delta$ values close to $-2m_e$. Table 6 lists favorable cases for $Q_{3b}(Z)$, while the most interesting cases for $Q_{3b}(1)$ and $Q_{3b}(0)$ are displayed in Table 7. The table shows that cases with $k = 1$ are less favorable than those with $k = 0$, because the expression for $k = 1$ contains in addition the negative quantity $-\Delta\widehat{B}_1$.

To summarize the above analysis, we have examined two- and three-body final states for a variety of nuclear processes in neutral atoms and fully or almost fully stripped nuclei. Kinematically, the most promising candidate for two-body reactions is the dysprosium-holmium case with $Q_{2a}(Z) \simeq -2.6$, while that for three-body reactions is the plutonium-americium case with $Q_{3a}(0) \simeq -1.6$.

Let us next return to the issue of partially stripped ions with k electrons. A complete analysis of all possible cases would require a numerical computation of the values of $B(k, Z)$ for all values of k and Z, a task beyond the scope of this paper.

${}^{A}_{Z+1}\mathrm{N}_2$	τ_{N_2}	${}^{A}_{Z}\mathrm{N}_1$	${}^{A}_{Z}\Delta$	ΔB_Z	$\Delta\widehat{B}_1$	$Q_{3b}(1)$	$Q_{3b}(0)$
${}^{191}_{78}\mathrm{Pt}$	2.9 d	${}^{191}_{77}\mathrm{Ir}$	-1000.0	16.0	2.6	-8.6	-6.0
${}^{122}_{54}\mathrm{Xe}$	20.1 h	${}^{122}_{53}\mathrm{I}$	-1000.0	9.0	1.6	-14.6	-13.0
${}^{236}_{93}\mathrm{Np}$	1.2×10^5 y	${}^{236}_{92}\mathrm{U}$	-984.0	21.6	3.4	-19.8	-16.4
${}^{67}_{31}\mathrm{Ga}$	3.3 d	${}^{67}_{30}\mathrm{Zn}$	-1001.1	4.0	0.8	-17.7	-16.9
${}^{83}_{37}\mathrm{Rb}$	86.2 d	${}^{83}_{36}\mathrm{Kr}$	-998.0	5.2	1.1	-19.9	-18.8
${}^{238}_{96}\mathrm{Cm}$	2.4 h	${}^{238}_{95}\mathrm{Am}$	-980.0	22.9	3.5	-22.6	-19.1
${}^{203}_{82}\mathrm{Pb}$	52.0 h	${}^{203}_{81}\mathrm{Tl}$	-974.0	17.4	2.7	-33.4	-30.7
${}^{220}_{90}\mathrm{Th}$	9.7 μs	${}^{220}_{89}\mathrm{Ac}$	-916.0	20.3	3.2	-88.9	-85.7
${}^{76}_{33}\mathrm{As}$	26.3 h	${}^{76}_{32}\mathrm{Ge}$	-923.0	4.4	0.9	-95.6	-94.7
${}^{250}_{100}\mathrm{Fm}$	30 m	${}^{250}_{99}\mathrm{Es}$	-900.0	24.7	4.0	-101.2	-97.3

Table 7: Smallest negative $Q_{3b}(1)$ and $Q_{3b}(0)$ values in keV for three-body reactions of type 3b.

Instead, we present here an analysis for a specific case, the dysprosium-holmium pair. Intermediate ionicities are of particular interest for this example not only because ${}^{A}_{Z}\Delta$ is small but also because the quantities $Q_{2a}(k)$ and $Q_{2b}(k+1)$ change sign as one passes from the neutral dysprosium atom to the fully stripped ion.

Several codes are available for computing binding energies. For the present case, we need the values of $B(k,67)$ and $B(k,66)$ for a broad range of k. Results for these binding energies obtained[2] using the code GRASP2 [20] are shown in Table 8. From them, we can obtain the values of ΔB_k and $\Delta\widehat{B}_k$, also shown in Table 8. Together with the observation that for the dysprosium-holmium system ${}^{A}_{Z}\Delta - \Delta B_0 \simeq -15.1$ keV, these results suffice to determine $Q_{2a}(k) \simeq -15.1\ \mathrm{keV} + \Delta B_k$, $Q_{2b}(k+1) = -Q_{2a}(k)$, and $Q_{3a}(k) \simeq -15.1\ \mathrm{keV} + \Delta\widehat{B}_k$. Note that the values of $Q_{3b}(k)$ are not interesting in the present context because they are dominated by the factor of $-2m_e$ appearing in Eq. (15).

The results for $Q_{2a}(k)$ are displayed in the last column of Table 8. Those for $Q_{2b}(k+1)$ follow by changing the sign. The table shows the existence of some kinematically favorable partially stripped cases. The most attractive one appears when $k = 27$, for which $Q_{2a}(27) \simeq -1.4$ keV, the smallest value yet found. For $k = 6$, a minimum in $Q_{2b}(7)$ of -4.5 keV is manifest.

[2]These results were computed and communicated to us by Farid Parpia.

n	$B(67-n,67)$	$B(66-n,66)$	ΔB_{66-n}	$\Delta\widehat{B}_{66-n}$	$Q_{2a}(66-n)$
66	65.2	0.0	65.2	0.0	50.1
65	129.2	63.2	66.0	2.0	50.9
64	145.0	125.1	19.9	4.1	4.8
63	160.4	140.3	20.1	4.7	5.0
62	175.5	155.3	20.2	5.1	5.1
61	190.1	169.8	20.3	5.7	5.2
60	203.6	184.0	19.6	6.1	4.5
59	216.7	197.0	19.7	6.6	4.6
58	229.5	209.7	19.8	7.0	4.7
57	242.0	222.1	19.9	7.4	4.8
56	247.6	234.2	13.4	7.8	-1.7
55	253.1	239.6	13.5	8.0	-1.6
54	258.4	244.9	13.5	8.2	-1.6
53	263.5	250.0	13.5	8.4	-1.6
52	268.3	254.9	13.4	8.6	-1.7
51	273.0	259.5	13.5	8.8	-1.6
50	277.6	264.1	13.5	8.9	-1.6
49	282.1	268.5	13.6	9.1	-1.5
48	286.2	272.8	13.4	9.3	-1.7
47	290.2	276.8	13.4	9.4	-1.7
46	294.1	280.6	13.5	9.6	-1.6
45	297.9	284.4	13.5	9.7	-1.6
44	301.5	288.0	13.5	9.9	-1.6
43	305.0	291.4	13.6	10.1	-1.5
42	308.3	294.7	13.6	10.3	-1.5
41	311.6	298.0	13.6	10.3	-1.5

Table 8: Binding energies and $Q_{2a}(k)$ values in keV for holmium and dysprosium ions (continued on next page).

n	$B(67-n,67)$	$B(66-n,66)$	ΔB_{66-n}	$\Delta\widehat{B}_{66-n}$	$Q_{2a}(66-n)$
40	314.7	301.1	13.6	10.5	-1.5
39	317.7	304.0	13.7	10.7	-1.4
38	319.5	306.9	12.6	10.8	-2.5
37	321.2	308.6	12.6	10.9	-2.5
36	322.8	310.3	12.5	10.9	-2.6
35	324.4	311.8	12.6	11.0	-2.5
34	325.8	313.3	12.5	11.1	-2.6
33	327.2	314.6	12.6	11.2	-2.5
32	328.5	315.9	12.6	11.3	-2.5
31	329.8	317.2	12.6	11.3	-2.5
30	331.0	318.5	12.5	11.3	-2.6
29	332.0	319.5	12.5	11.5	-2.6
28	333.0	320.5	12.5	11.5	-2.6
27	334.1	321.5	12.6	11.5	-2.5
26	335.0	322.4	12.6	11.7	-2.5
25	335.9	323.3	12.6	11.7	-2.5
24	336.7	324.1	12.6	11.8	-2.5
23	337.5	324.9	12.6	11.8	-2.5
22	338.3	325.7	12.6	11.8	-2.5
21	339.0	326.4	12.6	11.9	-2.5
20	339.5	327.0	12.5	12.0	-2.6
19	340.0	327.5	12.5	12.0	-2.6
18	340.4	327.9	12.5	12.1	-2.6
17	340.8	328.3	12.5	12.1	-2.6
16	341.2	328.7	12.5	12.1	-2.6
0	343.4	330.9	12.5	12.5	-2.6

Table 8: Binding energies and $Q_{2a}(k)$ values in keV for holmium and dysprosium ions (continued from previous page).

The values of $Q_{3a}(k)$ are not explicitly presented in the table because, as can be seen from the values of $\Delta\widehat{B}_k$, they are all negative. They have magnitudes decreasing monotonically with increasing k, down to the minimum of $Q_{3a}(66) \simeq -2.6$ keV already found. Indeed, the value $Q_{3a}(k) \simeq -2.6$ keV holds for $k \geq 48$. As might be expected from the relatively small size of the binding energies of the outermost electrons in a neutral atom, $Q_{3a}(k)$ changes little for k near Z. This means our earlier results for $k = Z$ have wider applicability than just to the neutral atoms. In principle, this could provide a means of accelerating effectively neutral atoms for the purposes of a null test.

3. Discussion

We have seen above that there are a number of candidate systems fulfilling the kinematic criteria for a null test with relatively small Q values. For a more definite experimental proposal, several facts should be borne in mind.

First, the atomic mass tables we have used [16] were originally published over fifteen years ago. Although in the intervening period more results have been obtained, to our knowledge there has been no analogous recent systematic compilation, as is required for the present analysis. In particular, this means that some of the Q values could be significantly less than those we have recorded above. For this reason, we have also listed systems with Q values that, although still relatively small, are considerably larger than the smallest ones found. Similarly, we have not attempted to estimate errors in the data presented.

Second, in distinguishing between two-body and three-body final states, we are assuming that the experiment can separate these two cases. If only the transmutation of one atomic species into another is detected, then for the test to be meaningful *both* relevant Q values must be negative.

Finally, in the above analysis we have assumed an ideal case for which the nuclides in question are in the ground state and have a long enough half life for an experiment to be performed. In particular, we have been primarily concerned with Q values rather than decay rates. Determining the latter is a much more involved task than

for the case $\mu \to \pi\nu$ mentioned in the introduction, and such a calculation must await a more definitive experimental proposal.

Let us briefly address the issue of the boost required for a null test with a given Q value. A reasonable guide to the boost needed can be obtained by approximating a generalization of the formula (2) for the threshold energy of the two-body decay $\mu \to \pi\nu$. Consider the general two-body null-test reaction $X_1 \to X_2\nu$, and denote by M_1 the mass of the parent body and $M_2 > M_1$ the mass of the daughter. Let $\bar{Q}$ represent the modulus of the Q value for this reaction, so that $M_2 = M_1 + \bar{Q}$. Then, we find

$$E_{\rm th}^2 = \frac{1}{4m^2}(\bar{Q}^2 + m^2)\left[(2M_1 + \bar{Q})^2 + m^2\right] \quad , \tag{16}$$

where m denotes the real and positive mass parameter for the neutrino.

It is certainly true that $m \ll M_1$. Moreover, $\bar{Q} \ll M_1$ for all reactions of interest. It is therefore a good approximation to write

$$E_{\rm th} \approx \frac{M_1\bar{Q}}{m}\left(1 + \frac{m^2}{\bar{Q}^2}\right)^{1/2} \quad . \tag{17}$$

If $m \ll \bar{Q}$, as is true for all processes considered above, the last factor in parentheses can be neglected. In the reactions of interest, M_1 represents the mass of a nucleus of baryon number A, so $M_1 \approx A$ in GeV. We finally obtain

$$m \approx \frac{\bar{Q}}{(\epsilon_{\rm th}/{\rm GeV})} \quad , \tag{18}$$

where $\epsilon_{\rm th}$ is the threshold energy per nucleon of the parent nucleus.

The most promising cases we have identifed so far have $\bar{Q}$ of about one keV. Plans at CERN and RHIC call for ion beams with energy of order 100 A GeV. Formula (18) shows that such beams are kinematically sensitive to spacelike neutrinos with mass parameter m on the order of 10 eV. This is competitive with current bounds obtained by other means.

Acknowledgments

We thank Farid Parpia for computing the binding energies presented in Table 8. We benefitted from communications with Walter Johnson, Bob Pollock, and Buford

Price. V.A.K. thanks the Aspen Center for Physics for hospitality while part of this investigation was performed. This work was supported in part by the United States Department of Energy under grant number DE-FG02-91ER40661.

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